Managers often search for ways to expedite project delivery times. A typical approach to achieve this goal is to add more resources. This strategy, however, does not always accomplish the desired result because not all tasks can be parallelized – some tasks must be completed sequentially. In accordance with the law of diminishing returns, net improvement to the total time diminishes with each newly added resource. Experienced project manager knows that every project has a point after which adding a new resource becomes counterproductive – this is called the "saturation point."

Our prior article illustrated that Amdahl formula used in parallel computing can be applied to calculate project time improvement. Since the original Amdahl formula does not take such factors as project overhead into consideration, one can only estimate the maximum time improvement (i.e., calculate an optimistic estimate). As new people are added to a project, new tasks are introduced. These new tasks often represent growing project overhead. In the prior article, we enhanced Amdahl’s formula to include project overhead to find the maximum time improvement.

Finding a project’s saturation point is significant, and in this article we take our discussion further. We analyze the relationship between the saturation point and the level of project parallelization. We also analyze the overall effect that overhead has on the saturation point. Finally, we provide a case study and the mathematical approach for calculating a saturation point for a specific scenario.

A Quick Reminder

As discussed in our prior article, the Amdahl formula was used for a rough estimate of a possible Total Project Time Improvement $I$

$$I(R) = \frac{1}{1 - P + \frac{P}{R}}$$

where $P$ is the percentage of Parallelized Tasks and $R$ is the number of Allocated Resources

The result from the above formula represents a theoretical scenario that is not affected by additional factors. In real life however, results are affected by the existence of project overhead. We proposed to derive a more precise value of $I$, based on the enhanced Amdahl formula

$$I(R) = \frac{1+O}{(1+O - P) + \frac{P}{R}}$$

where $O$ is a Project Overhead that consists of fixed and variable parts.
Case Study

We are going to take a look at projects in which not all tasks can be parallelized. For our case study let’s examine projects with overhead that has a fixed portion \( o \) and a variable portion that grows linearly—\( k \times R \), where \( k \) is a coefficient of the overhead per person, and \( R \) represents allocated resources. Therefore, Total Project Overhead is: \( O = o + k \times R \)

As discussed in the previous article, the Total Project Time Improvement \( I(R) \) time can be calculated by:

\[
I(R) = \frac{1 + o + k \times R}{(1 + o + k \times R - P) + \frac{P}{R}}
\]

Formula 1

In addition, this case study analyses include the following assumptions:

1. The number of project tasks that can be parallelized, overhead \( o \) and \( k \) are constants for the duration of the project, where \( 0 < P < 100, o > 0, k > 0 \).
2. The number of project tasks that can be parallelized is independent from number of resources (\( P \) is independent of \( R \)).
3. The number of project tasks that can be parallelized and project overhead are mutually independent (\( P \) is independent of \( O \) and \( O \) is independent of \( P \)).

The Saturation Point and Parallelization

To analyze a relationship between a project’s saturation point and Project Parallelization \( P \), let’s use Formula 1 and study the results of calculations for five projects that fit the case study’s assumptions. All five projects in this example have the same overhead. For example, all projects have a fixed overhead portion of 10 percent (\( o = 0.1 \)) and a variable overhead portion that grows linear \( k \times R \) with \( k \) equaling 50 basis points per each new resource (\( k = 0.005 \)). Let’s also assume that each project has a different number of tasks that can be parallelized. For example, for one project 99 percent of tasks can be parallelized, for another, 90 percent of tasks, and so on (i.e. \( P = \{0.99, 0.90, 0.80, 0.70, 0.50\} \)). We substitute \( o, k \) and \( P \) in our Formula 1 and for different values of \( R = \{5, 10, ..., 65\} \) for each project the results of calculations \( I(R) \) are displayed in the Table 1:
These results are graphed in Figure 1.

The above calculations demonstrate that a level of task parallelization has no impact on the saturation point. All five projects reached the same saturation point with the same value of $R = 16$. The $I(R)$ curve peaks at $R = 16$ and starts descending after that. In addition, Figure 1 demonstrates that there is a direct dependency between task parallelization and maximum Total Project Time Improvement $I(R)$; the more parallelization the greater is the Total Project Time Improvement $I(R)$.

In summary, for all five projects, adding resources beyond 16 people is counterproductive as the value of $I(R)$ starts to decline and the project delivery time gets longer.
Saturation Point and Overhead

To analyze the relationship between a project's saturation point and Total Project Overhead \( O \), let's take an example similar to the previous one: five projects with an overhead, where \( O = o + k \times R \). However, unlike the prior example, the Project Task Parallelization is the same for all projects, and the variable portion of overhead is different for each project. The Task Parallelization is equal to 90 percent \( (P = 0.9) \), and the Total Project Overhead values are calculated based on \( k = \{0.01, 0.005, 0.0025, 0.001, 0.0005\} \).

The results are shown in Table 2 and the corresponding graph is shown in Figure 2.

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<th>( R = 5 )</th>
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*Table 2. The Saturation Point and Overhead*
Table 2 shows that the saturation point is affected by overhead, or more specifically by its variable portion. After substituting values of $k = \{0.010, 0.0050, 0.0025, 0.0010, 0.0005\}$ into Formula 1, the five projects reach their saturation points with $12, 16, 22, 34$ and $48$ people respectively.

Figure 2 illustrates that there is an inverse relationship between the $O$ and $I(R)$. An increase of the Project Overhead leads to a decrease of Project Time Improvement $I(R)$. Overhead also affects the saturation point. Figure 2 demonstrates another inverse relationship between Project Overhead and a saturation point. Larger Project Overhead causes the saturation point to be reached with the smaller number of resources.

Based on these five sample projects, we demonstrated an intuitive conclusion: lower per-person overhead allows adding resources more effectively.

**Saturation Point and a Dose of Math**

Until now our conclusions were supported by samples of calculated values and curve trend analyses. However, we can validate our findings with strict mathematical calculations.

Not to overwhelm you with math, we provide a brief outline in the article, but you can view detailed calculations [on our web site](#).

From a mathematical point of view, the task of calculating a saturation point for a given project is a task of finding $R$ where $I(R)$ reaches its maximum. Our calculations have practical purpose and we are only dealing with people. Values of $R$ can only be whole numbers, so we are rounding results of our calculations. We are also going to analyze $I(R)$, for $R > 1$

If you recall, the first step in finding the maximum of a function is finding the critical points, followed by analyzing the value of the function at those points.

Let’s look again at our case study formula for Total Project Time Improvement:
\[ I(R) = \frac{1 + o + k \times R}{(1 + o + k \times R - P) + \frac{P}{R}} \]

For simplicity of notation, let’s introduce the following annotations: \( f(R) \) is a numerator, and \( g(R) \) is a denominator. Then, the formula for \( I(R) \) can be written as

\[ I(R) = \frac{f(R)}{g(R)} \]

To find critical points (e.g., minimums and maximums), we need to find the first derivative of \( I(R) \) and equate it to zero. To find a first derivative, we can use the quotient rule:

\[ I'(R) = \frac{f'(R) \times g(R) - f(R) \times g'(R)}{(g(R))^2} \]

and equate the first derivative of \( I(R) \) to zero:

\[ I'(R) = \frac{f'(R) \times g(R) - f(R) \times g'(R)}{(g(R))^2} = 0 \]

In order for a fraction to be equal to zero two rules must be preserved: a numerator is equal to zero and denominator is not. This means that we have to solve the following system of equations:

\[ \begin{cases} f'(R)g(R) - f(R)g'(R) = 0 \\ (g(R))^2 \neq 0 \end{cases} \]

We are going to concentrate on the numerator, and after performing a number of steps, we solve a quadratic equation and find the value of \( R \) where function \( I(R) \) reaches the maximum.

\[ R = 1 - \frac{\sqrt{k^2 + k \times (1 + o)}}{-k} \]

Formula 2

Substituting values of \( k = 0.005, \, o = 0.1 \) into the formula above, we calculate the value of \( R = 15.866 \); and this is fully in line with \( R = 16 \) answer received based on Table 1.

**Summary**

We can summarize our findings into three main conclusions:

- Project Parallelization \( P \) has no effect on the saturation point. Project Parallelization is directly proportional to Project Time Improvement—the larger the \( P \), the larger the value of \( I(R) \).

- Project Total Overhead \( O \) has an inverse proportionality affect on Project Time Improvement—the larger the \( O \), the smaller the values of \( I(R) \).

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• Project Total Overhead $O$ has an inverse proportionality affect on project saturation point –the larger the $O$, the smaller the values of $R$ at the saturation point.

**About the Authors**

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